Design and development of a ceiling-mounted workshop Measurement Positioning System for large-scale metrology

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Abstract

This paper presents a new ceiling-mounted workshop Measurement Positioning System (C-wMPS) compensating for many deficiencies shown by conventional metrology systems, especially on the possibility of task-oriented designing for coverage ability, measurement accuracy and efficiency. A hybrid calibration system consisting of a high-precision coordinate control field and standard lengths is developed and implemented for the C-wMPS, which can be designed concretely to provide both traceability and the ability of local accuracy enhancement. Layout optimization using a genetic algorithm based on grids is applied to design an appropriate layout of the system, therefore promotes the system's performance and reduce cost. An experiment carried out at the Guidance, Navigation and Control laboratory (GNC lab, 40×30×12m) validates the prominent characteristic of C-wMPS and the fitness of the new calibration system and layout optimization method.

Keywords: ceiling-mounted transmitter, coordinate control field, standard length constraints, layout optimization

1. Introduction

Large-scale collaborative digital manufacturing and assembly urgently requires precise, efficient and multi-tasking measurement technology in a unified coordinate system, which demands the measurement results be achieved in real-time [1-2]. The state-of-art measurement technology mostly focuses on centralized metrology systems (a typical representative is laser tracker), which partitioned working area into several sections to realize extra-large scale measurement, unavoidably bringing in accumulated error and inconsistent reference. Automation is also severely influenced by the transformations between stations [3-4]. Therefore, a representative distributed metrology system, indoor-GPS (iGPS), which is based on a rotary-laser automatic theodolite, arose under such conditions [5-7]. iGPS handles the expanding working area and the growing complexity of the tasks much more effectively.

The "around-space" arrangement of iGPS determined by each transmitter's measurement range shown in figure 1 provides relatively limited conditions to settle or adjust the whole metrology system. The $\pm 180°$ horizontal measurement range of each transmitter seems to be a fine ability but is largely wasted in engineering applications, therefore becomes much more costly to realize "full-coverage". Additionally, the limited vertical measurement range (approximately $\pm 30°$) restricts the height to install the transmitters. Although several researches and experiments have been carried out to basically design a proper layout for current distributed metrology system [8-9], the transmitters and measured objects positioning at the same height left no much choice for layout design in engineering. For most industrial measurement tasks, the operating distance along the horizontal direction is much larger than in the vertical direction. The rather large measuring "depth of field" increases the variation of positioning error, which is almost impossible to overcome by adjusting the system's arrangement. Also, light occlusion issues are very likely to happen, providing massive constraints for layout designing. Dynamic measurement is also difficult to realize [7,10]. In summary, these limitations lead to bad quality of iGPS that they usually lack the possibility of

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task-oriented designing.

This restricted quality not only affects the flexibility to design an outstanding layout for the transmitters but also confines the possibility to innovate the calibration method. Conventionally, scale bars are used as substantial constraints to adapt to the "around-space" arrangement as well as achieving calibration of the system. In the actual applications where measurement objects have already been arranged, the space to arrange the calibration system is usually confined. Intersection condition [11], as a major factor that affects accuracy, is mostly uncontrollable. Lower-quality calibration results will inevitably damage the measurement accuracy.

Therefore, we developed a new ceiling-mounted workshop Measurement Positioning System (C-wMPS) mainly to construct an integral measurement field with a unified coordinate reference, improving the possibility of task-oriented designing. Ceiling-mounted transmitter, as the essential component of the system, emits two laser planes downward to cover a conical-shaped measurement area as shown in figure 2. Arranging the transmitters appropriately on the ceiling can cover the dead zone of all transmitters by multi-station intersection. The working area, therefore, can be fully covered more easily. It can be seen that measuring "depth of field" is significantly reduced due to the different structure of transmitters because measuring directions are completely changed to adapt to common measurement tasks.

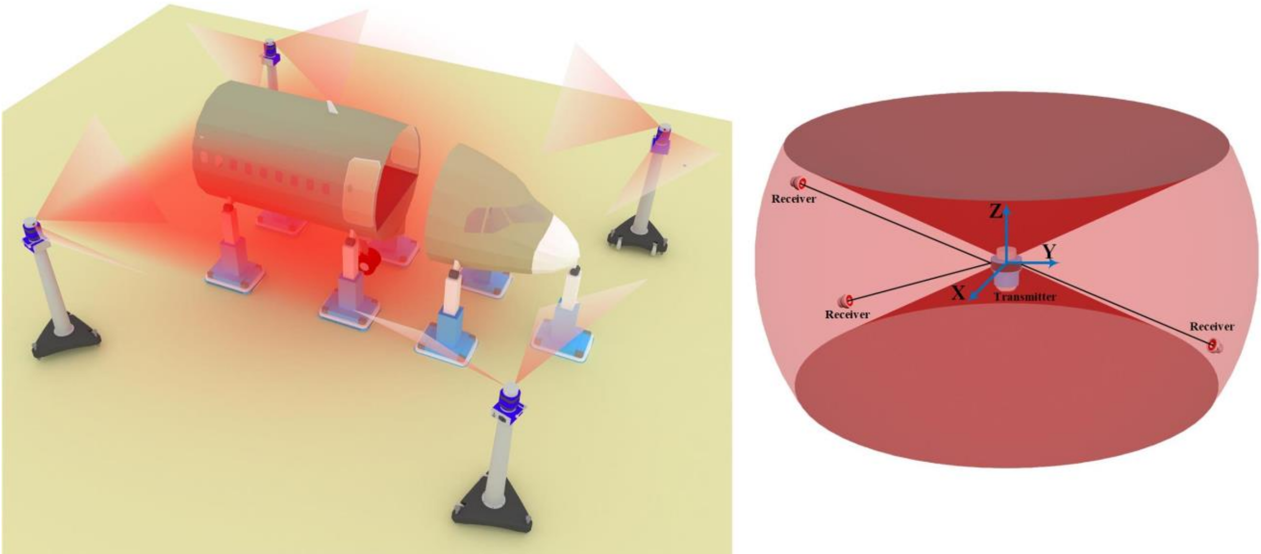

Figure 1: *Effective vertical measuring range of a transmitter (the dark-red area represents the dead zone)*

Improving the possibility of task-oriented designing primarily offers a sufficient condition to boost the performance of transmitters' orientation and layout design, which are two key technologies for C-wMPS. A major superiority is that the installation structure of C-wMPS naturally extends the space for calibration systems to be designed, therefore offers the possibility for a better calibration method. Therefore, a new calibration method is proposed to make full use of the advantages as well as to overcome defects of the conventional calibration method.

A coordinates control field constructs high accuracy 3-dimensional constraints, which can be well designed to fuse with the working area and conditions. It can be established by a multi-station laser tracker system to optimize trackers' angle measurement errors [12], providing high precision traceability. Characteristic points provided by the control field along with standard lengths as a local enhancement approach can be designed according to the specific demands of the task considerately, which will provide an optimal intersection condition [13] for the metrology system. The in-situ calibration system constructed before the measuring procedure also means that little human interference is needed to implement measurement. Efficiency is largely promoted.

Due to the unrestricted installation space, being able to design a task-oriented layout of transmitters is another profitable outcome of the newly designed system. The novel construction and features of C-wMPS mean that the accumulated experience for layout design may have little value. To make the best of the new system, coverage ability, positioning accuracy and total cost are considered comprehensively according to specific requirements and a balance between these factors.

Therefore, an accurate way with mathematical proof to optimize the layout aiming at the task is proposed. Genetic algorithm is applied to optimize the layout of the system. To promote efficiency and simplify calculations, the successive layout optimization problem is converted to a grid-based problem. Finite iterations are conducted to generate an optimized layout according to the target objective of the layout optimization procedure.

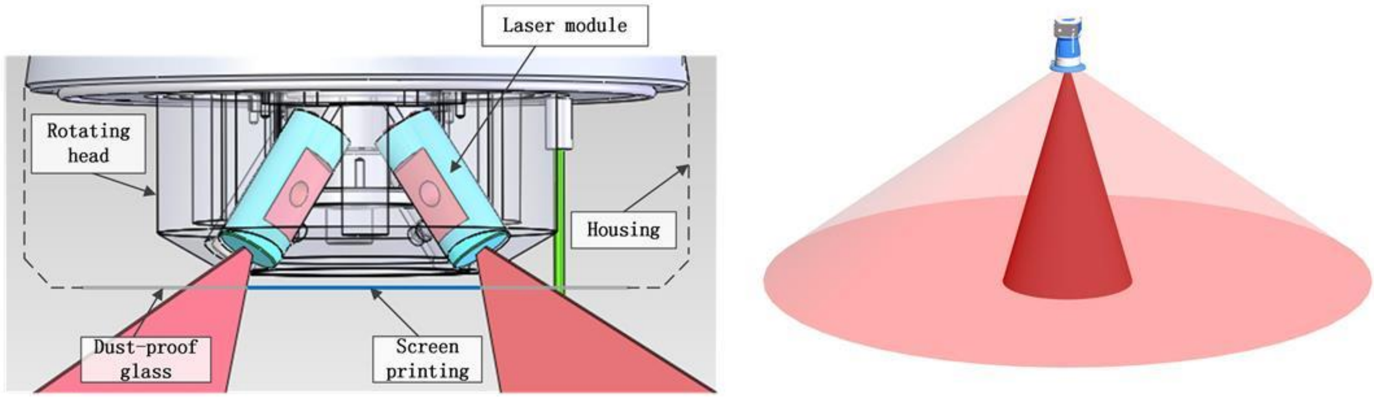

Figure 2: *Structure of a ceiling-mounted transmitter and its measuring range. The dark-red area represents the dead zone of a transmitter*

The remaining part of the paper is organized as follows: Section 2 introduces the measurement principle of the C-wMPS and the characteristics of a ceiling-mounted transmitter. Section 3 presents a novel calibration method mainly based on high precision coordinates control field with standard lengths as accuracy enhancement additionally. Section 4 proposes a genetic algorithm-based layout optimization method by dividing the available installation space into grids. In Section 5, experiments are presented to prove the validation for the above calibration and layout optimization methods. Finally, we present conclusions and potential future improvements for the system and technologies in Section 6.

2. Measurement Principle of C-wMPS

Ceiling-mounted workshop Measurement Positioning System uses optoelectric processing to transform time information into 2-dimensional angular information precisely. Transmitters working cooperatively can realize 3-dimensional coordinate measurement, and also, effectively extend the coverage ability of the system. Ceiling-mounted transmitters are the prime components of C-wMPS. A transmitter is mainly composed of a static base, laser modules and a precise rotating head. Tilted-angle and separation angle are two important parameters of a transmitter. The angles between two scanning laser planes and the rotary shaft are defined as the tilted-angles. The intersection angle of two scanning laser planes on the horizontal plane(XoY plane) is defined to be the separation angle of a transmitter. Since the basic purpose to develop C-wMPS is to measure the downward space so that the working area is completely uninterrupted, two laser planes with a 90° separation angle between them are installed on a precise rotary shaft at a uniform speed. The planar laser beams emitted by the modules is tilted by a small angle so that rotating laser beams can form a conical-shaped measurement area with a small dead zone, as shown in figure 2. Mathematical analysis, simulation and experiments results show that 15-20° will be the appropriate range of the tilted-angle to balance between the size of measuring range and measurement accuracy. The typical ceiling-mounted transmitter has an operating distance of 3-25 m, and the measurement area is a 53° cone projection in which 15° is the dead zone. Screen printing is applied as an effective way to limit the inner boundaries of the laser beams by shading each transmitter housing appropriately, which can prevent the inner laser beams from intersection as shown in figure 2.

According to conventional wisdom, the “around-space” configuration of a distributed metrology system is determined concerning the allocation of each transmitter’s measurement area, but inevitably a considerable percentage of them are wasted. Although it seems difficult to find an approach to avoid the dead zone dealing with C-wMPS, the essence of distributed metrology systems is that multiple transmitters installed according to a proper configuration can form a system which possesses the ability to cover the full working area. A typical example is present in figure 3, where the dead zone of one transmitter can be covered by two other transmitters in the system. In addition, measurement area of each transmitter is fully utilized due to the installation feature.

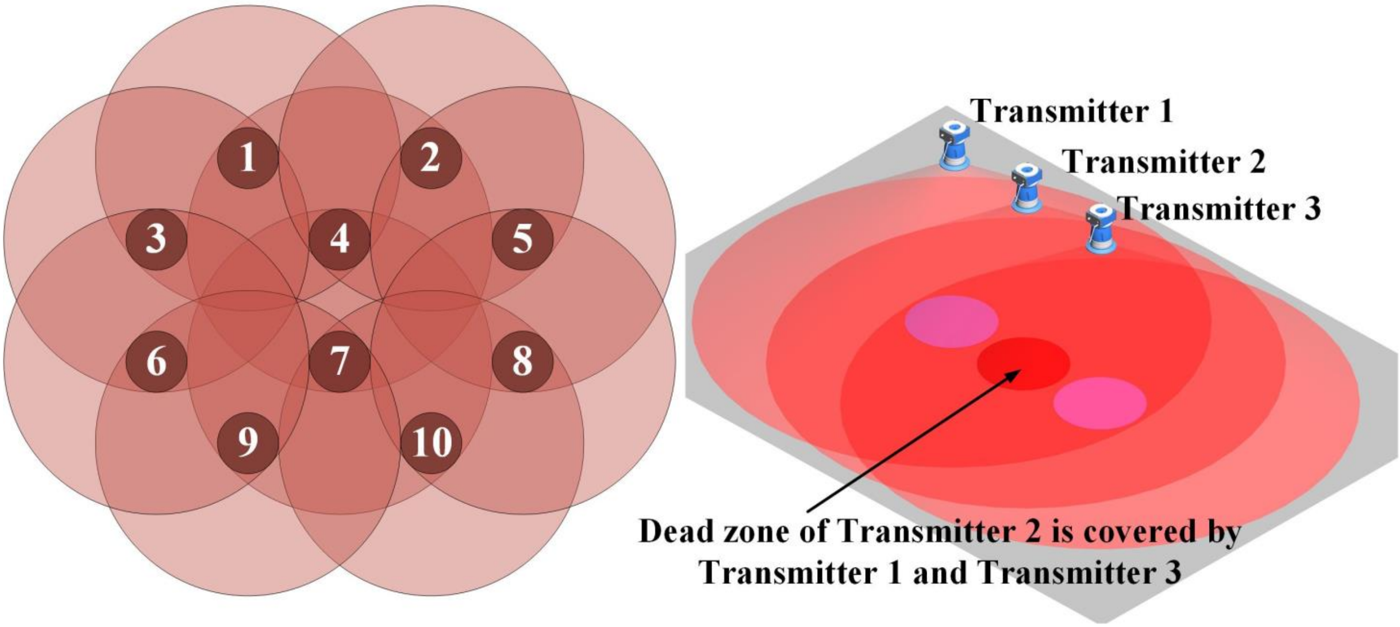

Figure 3: *A multi-station C-wMPS, dead zone of each transmitter can be covered by other transmitters in the system.*

To articulately explain the measurement principle of the system, define a local coordinate system (LCS) on a transmitter. The rotary axis is the Z-axis. The intersection point of the laser plane 1 and the Z-axis is the origin O. The X-axis is vertical to the Z-axis and on laser plane 1 at the initial time. Y-axis is defined conforming to the right-hand rule.

A series of synchronization signals are emitted by laser modules on the static base is used to identify the period of the system. When synchronization signals are received by a transmitter, it marks the initial time as t_0. The two scanning signals as t_1 and t_2. At the initial time, the i_{th} laser plane of the k_{th} transmitter can be described by plane parameters $a_{ki}b_{ki}c_{ki}d_{ki}$. The rotated angles of each laser plane are shown in figure 4.

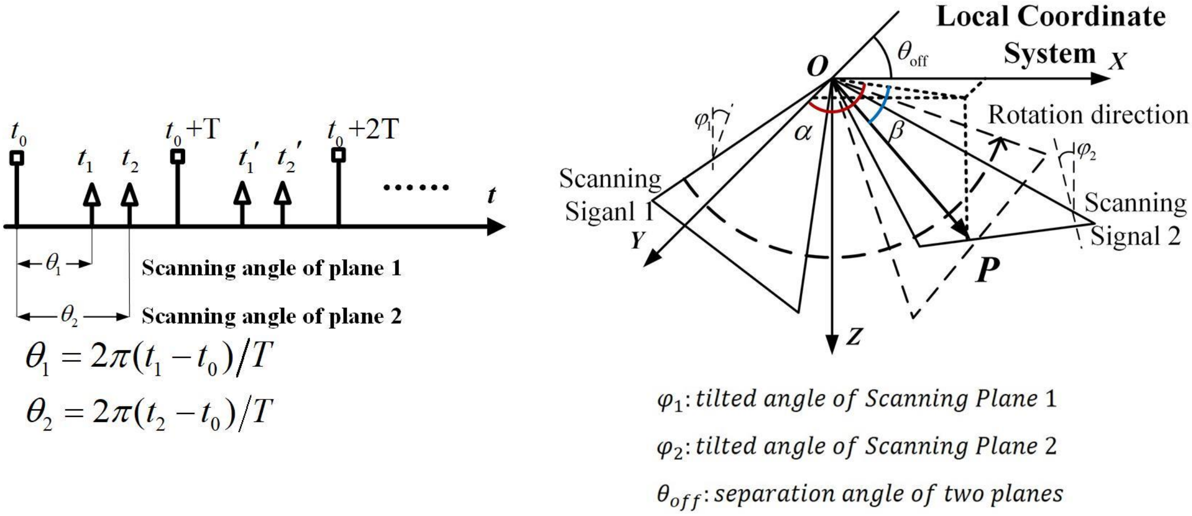

Figure 4: *Measurement principle of C-wMPS transforming time information to angular information*

When a receiver receives the scanning signal, the plane parameters of the relevant scanning laser plane are changed:

$$
\begin{bmatrix} a_{ki\theta} \\ b_{ki\theta} \\ c_{ki\theta} \\ d_{ki\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta_{ki} & -\sin\theta_{ki} & 0 & 0 \\ \sin\theta_{ki} & \cos\theta_{ki} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{ki} \\ b_{ki} \\ c_{ki} \\ d_{ki} \end{bmatrix} \tag{2.1}
$$

Since the coordinates of the receiver in the LCS are (x_l, y_l, z_l), the equations of the two laser planes must be satisfied:

$$
\begin{cases} a_{k1\theta}x_l + b_{k1\theta}y_l + c_{k1\theta}z_l + d_{k1\theta} = 0 \\ a_{k2\theta}x_l + b_{k2\theta}y_l + c_{k2\theta}z_l + d_{k2\theta} = 0 \end{cases} \tag{2.2}
$$

The ultimate demand of the measurement task is to achieve the coordinates of all targets in a unified global coordinate system (GCS). This entails building or selecting an appropriate coordinate system as the GCS and determining the transformations of the GCS to each transmitter's LCS. These transformations are represented by a rotation matrix R_k and a translation vector T_k. Summarizing the above statements, the coordinates of all targets can be calculated by implicit functions:

$$
\mathrm{F}(\theta_{ki}, C) = \begin{bmatrix} a_{ki} & b_{ki} & c_{ki} & d_{ki} \end{bmatrix} \begin{bmatrix} R(\theta_{ki})^T & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_k & T_k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = 0 \tag{2.3}
$$

Mounted on the ceiling, C-wMPS not only avoids interrupting the worksite but also retains enough room for the metrology system. The possibility of designing a proper distributed metrology system along with a calibration system is largely improved, which is rather difficult to achieve for most other systems. Another significant advantage C-wMPS brings about is the decrease of measuring “depth of field”, a definition resembles depth of field (DOF) in photogrammetry. The intrinsic measurement principle of the system is mainly angle intersection, which means the positioning accuracy will be seriously affected by the distance between targets and transmitters. C-wMPS considerably control the measurement accuracy by decreasing the measuring “depth of field”, that is, the practical operating distance. In other words, C-wMPS has changed the direction of the measuring “depth of field”, in order to adapt to most measurement tasks.

3. Orientation Parameter Calibration

3.1 Calibration method based on a precise coordinate control field with local accuracy enhancement

Performance of a distributed metrology system mainly depends on two elements: performance of a single measurement unit, and the accuracy of the orientation and position parameters of all the measurement units in the system [14]. For the latter, promoting the accuracy of the system’s calibration procedure will effectively make the system perform better.

The prevailing method for iGPS is to arrange scale bars vertically in front of the transmitters to construct some length constraints all over the working volume. The expectations are that there is enough clear space for scale bars to be arranged at many positions, and intersection angles between the transmitters and scale bars are acceptable. For common industrial application, these expectations are usually too rare to be achieved at the same time, resulting in a poor quality of the calibration procedure.

With the worksite largely emptied, it seems that the traditional method remains applicable for C-wMPS. However, the advantages of C-wMPS is not utilized. Thus, we deployed a hybrid calibration system for C-wMPS shown in figure 5.

Firstly, a coordinate control field consisting of a number of characteristic points can be well arranged in the working area according to worksite conditions. It can construct 3-dimensional coordinate constraints with high accuracy. Being designable according to the task requirements and traceable to the precise coordinates are two best features of a control field. Characteristic points providing geometrical constraints can be designed according to the specific requirements and the condition of the worksite. Once produced and fixed in-situ before measurement, the control field guarantees the traceability of accuracy during the whole measurement procedure. Also, it makes recalibration more convenient for the metrology system.

Additionally, placing several standard lengths in the measurement space as constraints can significantly improve local measurement accuracy. According to the experience for most measurement tasks, the accuracy requirements are rarely uniform but usually varies in different locations. Hence local accuracy enhancement is meaningful and necessary.

These two kinds of constraints constitute a hybrid calibration system. In figure 5, a series of characteristic points are installed, forming a control field. Relocating a laser tracker to several positions to form a measurement network can optimize the angle measurement accuracy by redundantly combining the high-accuracy Laser Interferometer Measurement (IFM), therefore improve the accuracy of the control field. In this way, the control field can have measurement uncertainty typically better than 0.02mm in 15m measurement range [15-17], which is far more accurate than measured by a single laser tracker. Because C-wMPS receivers and laser tracker reflectors share the same size, improving the accuracy of the control field can effectively improve the calibration accuracy.

There is hardly a strict demand for how to arrange such a control field. Basically, characteristic points surrounded the working area can noticeably improve the calibration accuracy. If the worksite condition sometimes cannot provide such good conditions, placing the characteristic points alongside the measurement targets is a fine alterable choice. It is also required that the control field should be fixed on stable positions such as on the load-bearing walls or on the floor. The stability of the control field can guarantee the correctness of the calibration procedure.

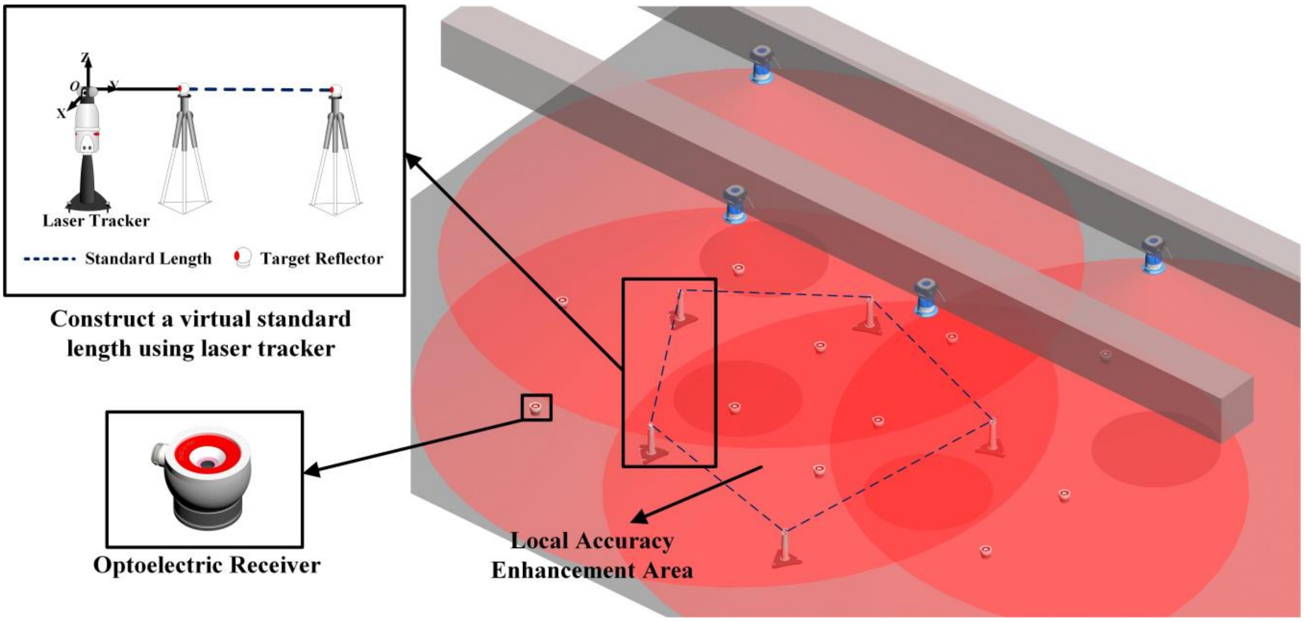

Figure 5: *Hybrid calibration system for C-wMPS*

Standard length constraints are also shown in figure 5 as a local accuracy enhancement approach. A most common way to bring in standard length constraints is to use scale bars. Besides costly and cumbersome, traceability is also difficult to ensure since the scale bars are usually removed once the calibration procedure is finished. To abandon substantial scale bars, we propose a more effective solution to construct some “virtual” standard lengths the same time the laser tracker forming a measurement network for the control field. Theoretically, two target reflectors fixed steadily in the measurement space can constitute a standard length similar to a scale bar. When measured roughly along the distance measurement direction of a laser tracker (IFM can be refined to micron level precision) [17], the angle measurement error will rarely affect the result. In this way, the distance between the two target reflectors can be precisely determined. It makes it possible and convenient to construct standard lengths almost everywhere in the worksite with accuracy traceable to laser wavelength of IFM.

It can be inferred by mathematical demonstration and proved by experiments that area near those standard lengths has relatively higher accuracy than the rest of the worksite. To make full use of the enhancement property, the specific locations to allocate standard lengths are determined according to the measurement task. This enhancement can be very useful, especially in the binding area for some assembly tasks.

In summary, since C-wMPS greatly improves the possibility of task-oriented designing, a hybrid calibration system is designed to promote the performance of the metrology system. The calibration system is mainly composed of a high-precision coordinate control field together with several standard lengths. While many of the advantages of the new calibration system have already been explained, it is necessary to mention that a slight adjustment to the measurement system will not invalidate the calibration procedure. Scalability and flexibility are both improved by the new method.

3.2 Detailed calibration procedure and optimization algorithm

When a receiver receives the scanning signal from a transmitter, the relevant laser plane rotates and the parameters of the laser plane transfer according to (2.2). Transforming the plane parameters from LCS to GCS:

$$[a_{gki\theta} \quad b_{gki\theta} \quad c_{gki\theta} \quad d_{gki\theta}] = [a_{gki} \quad b_{gki} \quad c_{gki} \quad d_{gki}] \begin{bmatrix} R_k^{-1} & T_k^{-1} \\ 0 & 1 \end{bmatrix} \tag{3.1}$$

Suppose that a control point $P(x_m, y_m, z_m)$ in the control field is being scanned. The distance between the control point and the current laser plane can be determined as:

$$d_{mki} = \left\| [a_{gki\theta} \quad b_{gki\theta} \quad c_{gki\theta} \quad d_{gki\theta}] \quad \begin{bmatrix} R_k & T_k \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} x_m \\ y_m \\ z_m \\ 1 \end{bmatrix} \right\|_2 \tag{3.2}$$

The above equation can be described as a laser plane constraint equation [16]. R_k and T_k represent the rotation matrix and translation vector, respectively, from the GCS to the k_{th} transmitter's LCS.

Dealing with standard lengths in the system, target balls on both ends of a standard length will receive signals from transmitters. On one hand, these endpoints are measured by multi-station laser trackers before the calibration process, thus they can be treated as common control points in the field so that laser plane constraint equations as (3.1) can still be constructed. On the other hand, a standard length will bring in a length constraint to the calibration process. To calculate the length according to the measurement of the wMPS, assume that the coordinates of the endpoints are unknown. If the measured coordinates of the endpoints are $P_{r1}(x_{r1}, y_{r1}, z_{r1})$ and $P_{r2}(x_{r2}, y_{r2}, z_{r2})$, laser plane constraint equations can be constructed:

$$d_{rjki} = \left\| [a_{gki\theta} \quad b_{gki\theta} \quad c_{gki\theta} \quad d_{gki\theta}] \quad \begin{bmatrix} R_k & T_k \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} x_{rj} \\ y_{rj} \\ z_{rj} \\ 1 \end{bmatrix} \right\|_2 \quad (j = 1,2) \tag{3.3}$$

The above constraint equations show a similar format as (3.2), except that (x_{rj}, y_{rj}, z_{rj}) is an unknown coordinate set. Differences between the length calculated from the measurement result of the wMPS and the calibrated length can be used to construct a length constraint:

$$\Delta l_j = \sqrt{(x_{k1} - x_{k2})^2 + (y_{k1} - y_{k2})^2 + (z_{k1} - z_{k2})^2} - L_{ck} \tag{3.4}$$

With these constraints constructed as above, R_k and T_k can be determined. Furthermore, when using Euler angles (θ, ψ, γ) to express the rotation matrix as shown in (3.5), the orthogonality of the matrix indicates that constraint equations as in (3.6) must be satisfied:

$$\mathrm{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} \cos\gamma\cos\psi & \cos\psi\sin\theta\sin\gamma - \cos\theta\cos\psi & \sin\theta\sin\psi + \cos\theta\cos\psi\sin\gamma \\ \cos\gamma\sin\psi & \cos\theta\cos\psi + \sin\theta\sin\gamma\sin\psi & \cos\theta\sin\gamma\sin\psi - \cos\psi\sin\theta \\ -\sin\gamma & \cos\gamma\sin\theta & \cos\theta\cos\gamma \end{bmatrix} \tag{3.5}$$

$$f_j = r_{p1}r_{q1} + r_{p2}r_{q2} + r_{p3}r_{q3} = \begin{cases} 0 & p \neq q \\ 1 & p = q \end{cases} \tag{3.6}$$

The number of constraint equations as well as unknown parameters can be decided based on the numbers of control points, reference bars, and transmitters. When the number of constraint equations is not less than the number of unknown parameters, the objective function can be formulated:

$$\mathrm{F} = \sum_{m=1}^{M}\sum_{k=1}^{N}\sum_{i=1}^{2} d_{mki}^2 + \lambda_1 \sum_{j=1}^{T} \left(\Delta l_j\right)^2 + \lambda_2 \sum_{n=1}^{6} \begin{cases} f_j^2 & p \neq q \\ \left(f_j - 1\right)^2 & p = q \end{cases} \tag{3.7}$$

λ_1, λ_2 are weight parameters of a different part of the objective function. λ_1 is usually greater than 1 since the standard length is a more accurate constraint than control point coordinates. λ_2 is a much greater to make sure that the rotation matrix is orthogonal.

It is necessary to ensure that each receiver can receive scanning signals from at least two different transmitters. The Levenberg-Marquardt algorithm [18] is chosen to be the optimization algorithm for Eq. 3.7. The optimization problem can be solved and the orientation for each transmitter is then determined. In this way, the orientation parameter calibration of the system is completed.

4. Layout Optimization Using Grid-Based Genetic Algorithm

4.1 Mathematical Model Of The Optimization Problem In C-wMPS

C-wMPS offers a better opportunity to design a proper layout, especially for complicated measurement tasks. Typical configurations of some common distributed metrology systems have been discussed and verified by experiments [11-13,19]. The dominant contradiction of layout optimization is between the high expectations of the tasks and the confined space to arrange the distributed metrology system. Also, conventional systems such as iGPS are easily affected by perturbations in-situ. Changes in the task or the condition often mean that an entirely different layout needs to be designed because of additional light occlusion issues and the accuracy variations. C-wMPS breaks the contradiction by changing its installation mode so that the worksite and the metrology system installation area are separated and “non-interference”. As for the perturbation problem, C-wMPS with a smaller measurement “depth of field” will remain workable when slight regulations occur. The layout is quite stable once designed, and only need adjustment when dramatic changes are happening.

Yet another factor which cannot be ignored is that installation mode of C-wMPS is much more inconvenient than most other systems. Installation locations for ceiling-mounted transmitters need to be produced in advanced. Once determined, it would be extremely troublesome to exploit new locations. For the matter of that, designing a layout for C-wMPS is more complicated and challengeable. Available installation locations also need to be considered carefully. We proposed a mathematical model for the layout optimization problem. Instead of merely concerning about the system’s accuracy in early study, our model proposed three principles to be considered when evaluating the system’s performance.

Coverage ability of a metrology system is essential to ensure the task is performable. Besides the measurement range of the each transmitter combined together, light occlusion issues caused by the measured object and other equipment in the worksite will influence the measuring range. Positioning accuracy is the key approach to promote the performance of the system. Total cost is a rather specific problem, which roughly demands the system to be under budget and each transmitter to have an acceptable utilization ratio. A mathematical model is built considering the principles mentioned above in (4.1):

$$F_{obj}(x) = K_1 O_1 + K_2 O_2 + K_3 O_3 \tag{4.1}$$

K_1, K_2, K_3 in the function are the weights and should satisfy:

$$\sum_{i=1}^{3} K_i = 1 \tag{4.2}$$

O_1, O_2 and O_3 are parts of the objective function describing the coverage ability, positioning accuracy and total cost of the system respectively. These parts will be specified later.

The optimization goal is to minimize the function $F_{obj}(x)$. x contains every essential parameter, including the transmitters’ position and orientation. The empirical way to assign the weights is that measuring range usually shares a high-priority because it directly affect the feasibility of the task, so that K_1 is usually much bigger than K_2, K_3. It should be noted that there is no rigid conventions for how to assign the weights. Weights can change according to the concrete requirements of the task. It can also be allocated dynamically when it behaves worse than expected [20]. There is no clear evidence that the objective function must be declared in such a way. On the contrary, with full consideration of the principles of optimization, objective function can be constructed under the specific requirements of the task or be regulated in the circumstances.

4.2 Detailed Principles For Layout Optimization

Before we describe the principles in detail, it should be noticed that these principles is not entirely independent. Coverage ability and positioning accuracy are in close connection with the system cost. One obvious example is that a better performance in coverage ability and positioning accuracy can make it possible to decrease the number of transmitters to control the total cost of the system. In turn, assessing the system’s total cost will expose those transmitters with a relatively lower utilization ratio. These transmitters have a modest effect on the system's function, so they are more likely to be removed.

4.2.1 Coverage Ability

Coverage ability is the key request of a measurement task. Approximately 80%-90% of the demanded measurement space should be covered by the system, given that the fringe area of the region is likely to be meaningless for the task. Since light occlusion issues have been weakened by the feature of C-wMPS to some extent, they have to be considered more concretely.

Dealing with light occlusion issues, we quantify the geometry information of the equipment referring to the concept of stereolithography (stl) file format [21], which uses a set of spatial triangles to mesh an entity. The light occlusion issues hence become a judging problem of whether the measuring vector will collide the mesh representing the equipment. Tomas Moller et al. [22] presented a quick approach to determine whether a ray intersects a triangle. Compared to the old model using several cuboids to represent the equipment approximately, the vector-mesh collision model for light occlusion sacrifices efficiency to promote the operability and exactitude of the optimization process.

The objective function of measuring range can be defined:

$$O_1 = 1 - \frac{n_m}{n} \tag{4.1}$$

In this function, n is the total number of targets sampled from the measured space, and n_m is the number of targets that can be measured by the current layout of transmitters. In general, the value of the objective function is expected to be no more than 0.2, or the layout may be invalid.

4.2.2 Positioning Accuracy

In most cases, a measurement task requires that positioning accuracy is constrained in a certain range. Improve positioning accuracy is a significant way to optimize the metrology system. When designing a layout, only angle measurement errors are taken into consideration, while coordinate transformation errors [14] and other systematic errors are ignored.

The evaluation of the positioning accuracy is treated as a simulation process. When the target coordinates $\mathrm{P}(x_i, y_i, z_i)$ and orientation information R_k, T_k of all transmitters are given. A virtual measurement process can generate the measurement results as well as the positioning uncertainty for each target. The measurement model in a virtual measurement process is similar to an actual measurement system as shown in (2.4). The relationship between the angle measurement uncertainty and coordinate uncertainty can be established according to the measurement principle:

$$u_p = \frac{dP}{dC} u_c \left(\frac{dP}{dC}\right)^T \tag{4.2}$$

where u_c represents the covariance matrix of angle measurement and dP/dC is the sensitivity matrix that can be calculated by the implicit differentiation method. u_p represents the covariance matrix of coordinates, which can be expressed by:

$$u_p = \begin{bmatrix} u^2(x) & u(x,y) & u(x,z) \\ u(y,x) & u^2(y) & u(y,z) \\ u(z,x) & u(z,y) & u^2(z) \end{bmatrix} \tag{4.3}$$

The covariance matrix clearly represents the uncertainty distribution information, which involves both magnitude and direction. The deficiency is that the covariance matrix is difficult to compare with the requirements of the task. An intuitive term named GDOP (Geometric dilution of precision) introduced in the area of GPS is used to evaluate distributed metrology system's precision property [23]. The function O_2 is constructed as follows, where GDOP_m is the maximum permissible GDOP of the task and n_m is the number of targets that are able to be measured:

$$O_2 = 1 - \frac{1}{n_m} \sum_{i=1}^{n_m} \begin{cases} 1, & \mathrm{GDOP}_i \le \mathrm{GDOP}_m \\ \dfrac{\mathrm{GDOP}_m}{\mathrm{GDOP}_i}, & \mathrm{GDOP}_i > \mathrm{GDOP}_m \end{cases} \tag{4.4}$$

When each of the targets' positioning accuracy satisfies the task's demand, the value of the objective function will be 0. Conversely, if the value of the function approaches 1, it is very likely that the present layout is not suitable for the task.

4.2.3 Total Cost

Define the utilization ratio of a transmitter:

$$f_r = \frac{n_m}{n} \tag{4.5}$$

In this function, n_m is the number of targets that the transmitter can scan, and n is the number of all targets in the working area. In this way, a transmitter with a bigger f_r in the system is defined more "useful".

Therefore, the objective function of total cost of the system can be defined as:

$$O_3 = \begin{cases} 1, & m > m_{max} \\ 1 - \frac{1}{m}\sum_{i=1}^{m} f_{ri}, & m \le m_{max} \end{cases} \tag{4.6}$$

where f_{ri} represents the i_{th} transmitter's utilization ration and m_{max} is the maximum number of transmitters the task can permit.

Once the total cost of the system is under budget, the utilization ratio should be considered. A smaller function value means the satisfaction of the task requirements and the high utilization ratio of the present layout.

4.3 Optimization Method Based On Genetic Algorithm

The mathematical model for layout optimization for a C-wMPS we proposed is aimed to optimize a rather complex discrete objective function. Algorithms based on differential of the function is no longer workable. Meta-heuristic algorithms are powerful methods for solving optimization problems [24]. The genetic algorithm is a typical meta-heuristic algorithm introduced by Holland in 1975 [25]. The optimization process based on this algorithm is produced by a flowchart in figure 6.

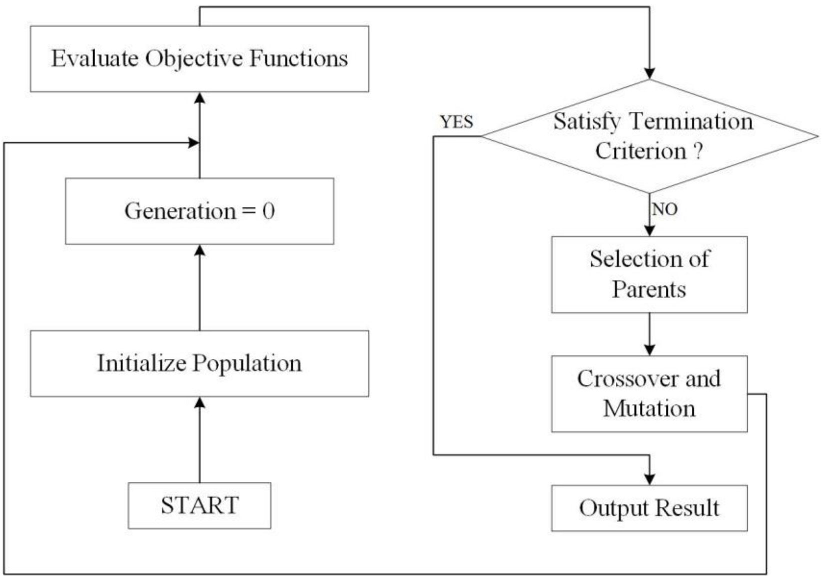

Figure 6*: Flowchart for genetic algorithm*

Encoding is a fundamental procedure in genetic algorithm. Crossover and mutation are conducted on the basis of the encoding options. One important thing to consider is the discrete encoding model of C-wMPS. The installation locations for ceiling-mounted transmitters need to be produced on the ceiling in advance and will be difficult to regulate thereafter. Therefore, the available locations is discrete. After a thorough consideration of the task, an interval was determined to divide the entire ceiling into a uniform grid for convenience of producing the installation locations in advance. Numbers are given to the grid nodes to represent the installation locations simply.

We also pay attention to the alterable orientation of ceiling-mounted transmitters before determining the encoding model. Being different from the position, the orientation of a ceiling-mounted transmitter can change in a predefined range. This is one of the main differences between ceiling-mounted transmitter and wMPS transmitter. The latter one is mostly installed on a tripod and perpendicular to the floor.

For the discrete encoding model, choose a simple encoding options which set the value corresponding to a grid node to 1 if there's a transmitter installed; otherwise the value remains 0. Dealing with the alterable orientation model, the continuous Euler angle expressing the orientation are reasonably discretized to reduce the computing time but can still provide enough available solutions for the problem.

Take an example that initially the ceiling was divided into 4×6 uniform grids and 3 transmitters are installed, an individual can be expressed as:

Table 1 Representation of an individual

1	2	3	······	24	θ_1	γ_1	θ_2	γ_2	θ_3	γ_3	······
0	1	0	······	0	0°	0°	0°	0°	0°	0°	······

Length of an individual is $24 + 2 \times 24$. Because there are only 3 transmitters installed, values corresponding to the last $2 \times (24 - 3)$ bits of the individual are set to NULL. The number of the transmitters installed is not explicitly involved in the encoding result, but can easily be calculated. As a result, all input parameters are covered by the encoding model and the objective function value can be calculate. Possibilities are generated to guide the crossover, mutation and selection procedure:

$$P_i = \frac{F_{obj}(x_i)}{\sum_{i=1}^{t} F_{obj}(x_i)} \tag{4.9}$$

where t represents the population of a generation and P_i represents the possibility for the i_{th} individual. Choosing a reasonable t is quite an inconclusive question that should be implement based on the complexity of the task and accumulated experience.

After several iterations, the latest generation is expected to satisfy the demand of the optimization problem. Then the individual with the best fitness, in other words, the minimized function value, will be selected as the optimal individual of the problem. Decoding the individual to obtain the optimized layout for the transmitters.

5. Experiments in GNC Lab

An experiment was carried out to prove the feasibility of the new calibration method and layout optimization for C-wMPS. The GNC Full Physical Simulation Laboratory (GNC lab) in the National Aerospace Test Base (Tianjin) demands to realize real-time measurement of a simulated spacecraft moving in the entire space. The C-wMPS was applied to the GNC lab because of its parallel measurement ability, large measuring range, and high precision. Most importantly, the installation feature of ceiling-mounted transmitters ensures that the working area is not disturbed by transmitters or any related equipment, which is almost impossible be achieved by other metrology systems.

To satisfy the requirements of the measurement task, designing a good layout of C-wMPS is of great importance.

5.1 Designing a Layout for C-wMPS in GNC Lab

Table 2 Specific requirements of the task

Requirements of the System	Parameter
Measurement Space	$40 \times 30 \times 3\mathrm{m}$ above the experiment platform
Measurement Accuracy	MPE: $\pm 0.5\mathrm{mm}$
The Total Cost	No more than 10 transmitters
Installation Positions	The ceiling of GNC Lab is 12m away from the experiment platform(working area $40 \times 30 \times 12\mathrm{m}$)

The Simulation Experiment Platform in the GNC lab has a size of 42m×32m. In an experiment, it was required that the 3-dimensional coordinates of any target on the platform or within 3 m above the platform can be measured with a demanded accuracy tolerance. The specific requirements of the measurement task are listed in Table 2.

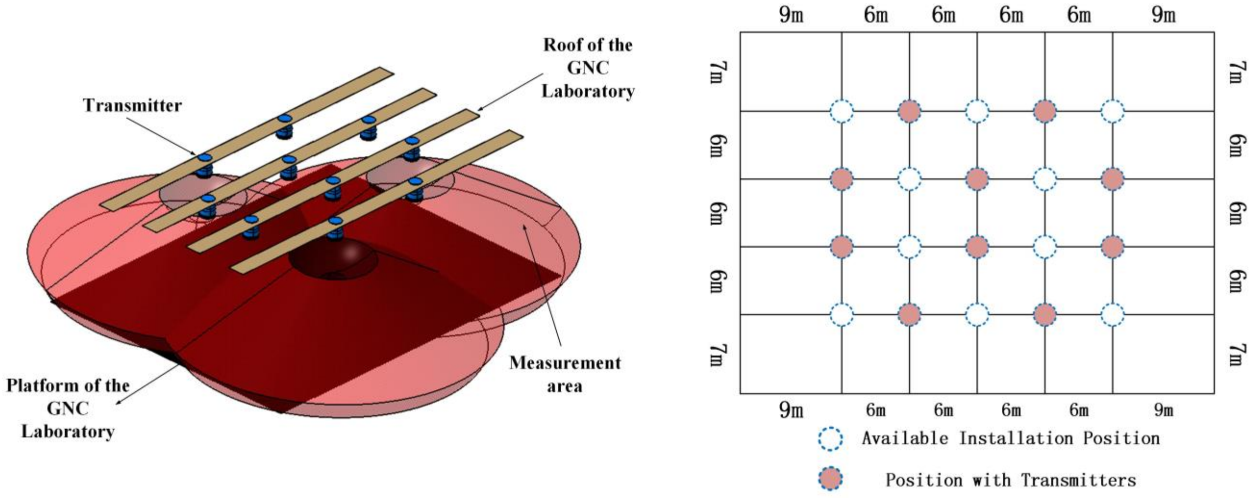

Figure 7*: Optimized layout of C-wMPS in GNC Laboratory*

On the ceiling 12m above the platform in the laboratory, we designed a 6m-interval grid which has 20 installation locations. The interval is mainly determined by the height of the ceiling and the measuring range of the ceiling-mounted transmitter. The grid and grid nodes are shown in figure 7. With the aim of covering the whole measuring space of the task required, 3 planes parallel to the platform were selected to perform the simulation, which were respectively 0 m, 1 m and 3 m above the platform. 3813 targets were sampled in each plane at 1 m intervals to check the performance of the system. Since the available positions for transmitters are determined, the fixtures on each location allow the horizontal angle to be adjusted within $0°\sim360°$ and the vertical angle $165°\sim180°$.

Table 3 Orientation for 10 transmitters represented by Euler angles

Transmitters	Horizontal Angle/°	Vertical Angle/°
1	225	170
2	135	175
3	225	170
4	180	175
5	0	175
6	45	175
7	0	175
8	45	170
9	0	170
10	45	175

In practical, the optimization procedure started with an initial generation of 10 individuals and took 53 iterations to generate the final layout with 10 transmitters installed. After the optimization, the position arrangement is shown as red circles in figure 7 and the orientation angles describing the horizontal and vertical angle of the rotary axis are shown in Table 3. Simulations results shown in figure 8 verified the performance of the system is able to satisfy the task's requirements. Although some marginal area remains unmeasurable (represented by white color in figure 8), the following experiments carried out in the GNC Lab have proved them insignificant. The measurement uncertainty is less than 0.5 mm, which is significantly decreased compared to the initialized layout before optimization and satisfies the requirements of the task.

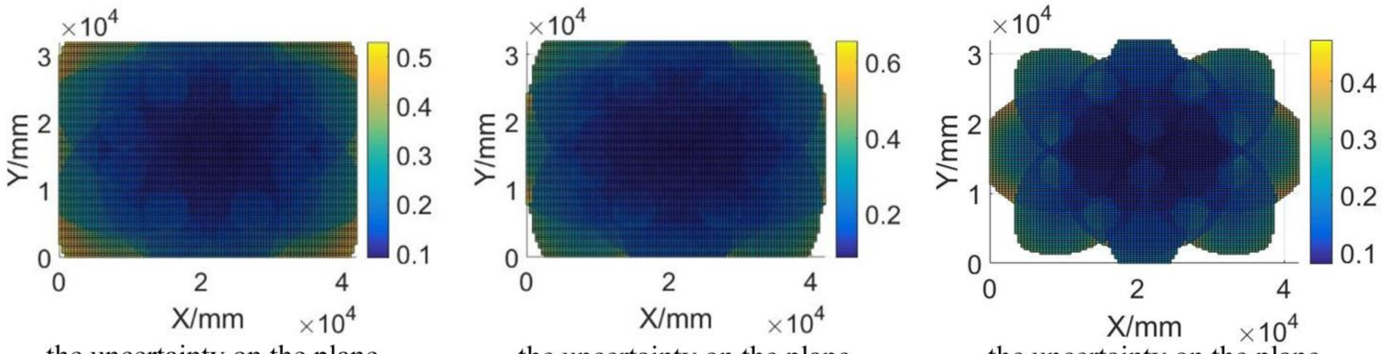

Figure 8*: Simulation results for C-wMPS of the optimized layout*

5.2 Verification

GNC Lab put forward a rather challenging measurement task since the work site is extremely larger than usual. Meanwhile, the upper limit of measurement accuracy is very strict considering such a large scale and low budget. C-wMPS with the optimized layout shown in figure 7 and Table 3 has been applied in the GNC Full Physical Simulation Laboratory. Acceptance test was conducted using the system. Also, we proposed an experiment to verify the property of the new calibration method mentioned in section 3.

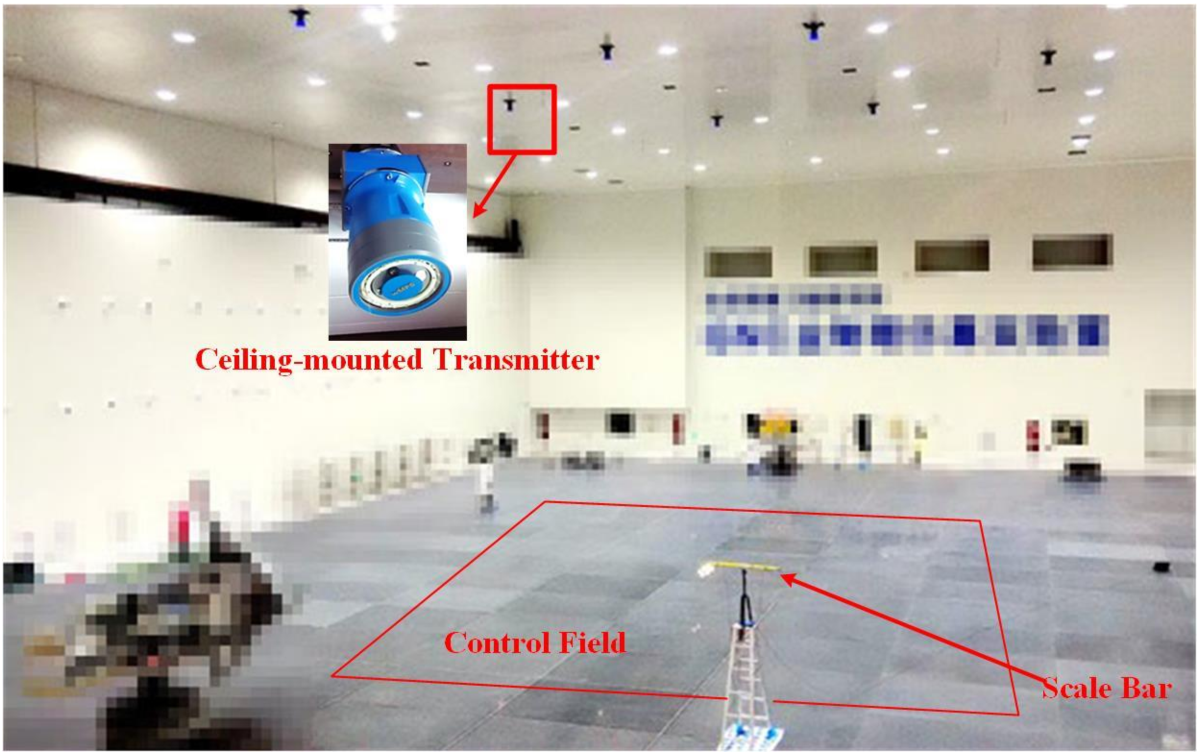

Figure 9*: Metrology system and working area in GNC Laboratory*

After installing all the transmitters and the relevant equipment correctly, we have fixed 50 points on the floor roughly uniform to construct a control field. A multi-station laser tracker system was used to measure the coordinates of these control points. At the same time, 10 of these control points in the middle area where simulated spacecraft usually go past were chosen together with several scale bars to form 8 standard lengths measured by the method proposed in Section 3. The standard lengths were used to verify the local enhancement ability of the calibration method. The control field together with the standard lengths forms the calibration system of the experiment.

With a reasonable calibration result, the measurement system satisfied all of the preparatory work. To verify the performance of the system, a scale bar with target reflectors fixed on both ends was brought into the working area. The calibrated length of the scale bar was 1652.11 mm. It was moved to 84 different positions approximately 1.4-2.0m above the platform in the working area, and the lengths of the scale bar were measured by the metrology system. Part of the deviations between the measured lengths and the calibrated length are shown in figure 10. The maximum deviation is 0.21mm. It can be seen from the figure that positions near the standard lengths constructed in the calibration system (marked area in figure 10) actually have a relatively higher measurement accuracy, which shows the capability of the calibration method.

To further prove that the system can satisfy the requirements of the measurement task, more than 400 target points were distributed in the worksite with heights from 0.5 m to 2.0 m in a series of experiments. A multi-station laser tracker was used to measure these target points as to provide the reference coordinates. Then the target points were measured by C-wMPS. Part of the deviation results is shown in figure 11. The local accuracy enhancement verification resembles the previous one. The first 15 test points (marked area in figure 11) clearly have relatively lower deviations because these targets are located in the middle area where measurement accuracy is enhanced by standard lengths.

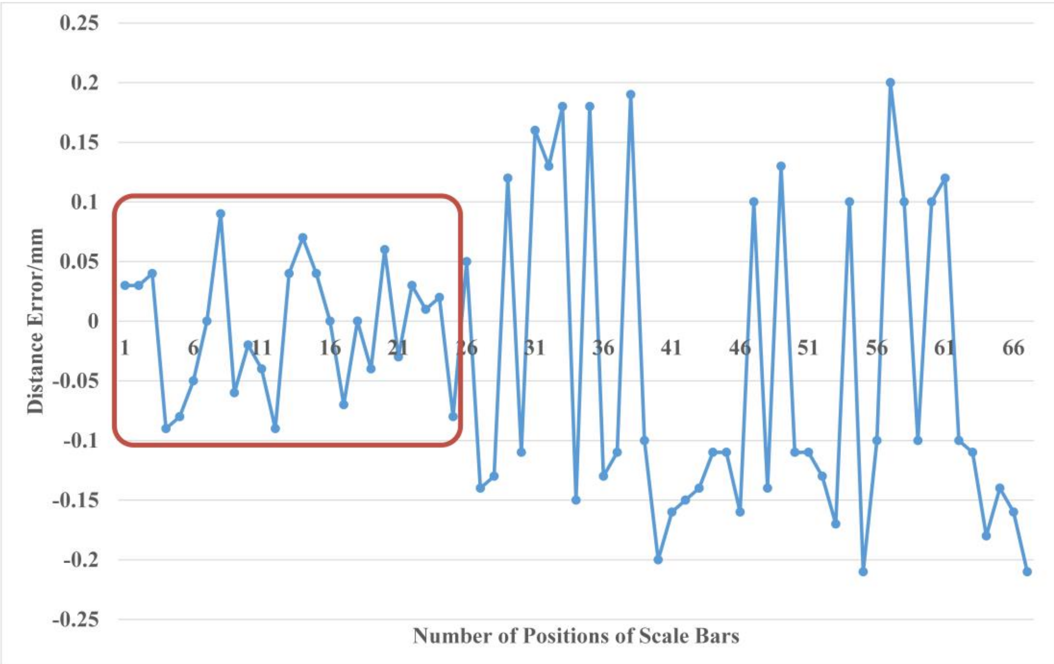

***Figure 10**: Part of the acceptance test results using scale bars*

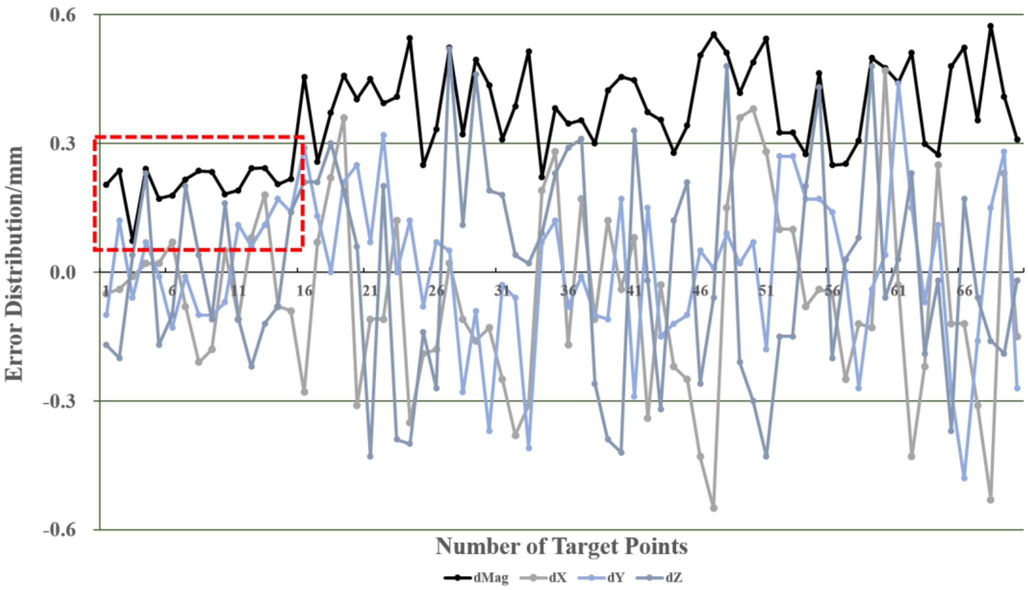

Figure 11: *Part of the acceptance test results for target coordinates*

The complete verification experiment results show some testing targets present lower-than-expected deviations, causing the maximum deviation reach -0.72mm. But more than 91% of the testing targets have satisfied the task requirements, which is a privilege for such an extra-large scale task. More to the point, almost all the bad-performing targets are located at the marginal area, where for most measurement tasks has little significance. Besides, the system's layout and calibration system can be easily regulated when the marginal area is used to satisfy the adjustment of the measurement tasks.

6. Conclusion

A ceiling-mounted transmitter workshop Measurement Positioning System (C-wMPS) has been designed to improve the performance of current distributed metrology system. Possibility of task-oriented designing is largely improved for the metrology system as well as the whole measurement process. We then present calibration and layout optimization methods to make the best of the highly designable feature of the system. The calibration system is composed of a high precision coordinate control field and several virtual standard lengths. The control field is highly designable according to the task requirements and ensures the accuracy traceability of the calibration procedure. Standard lengths were brought into the system to enhance the accuracy in a local area. Furthermore, the novel calibration method reduces the level of manual input and therefore promotes efficiency and automation of the measurement process. The flexibility to design a better layout is another key advantage of C-wMPS. A layout optimization method that considers coverage ability, positioning accuracy and the system cost is proposed based on genetic algorithm to make it possible to design a suitable layout for a specific measurement task, which needs few accumulated experience. Experiments in GNC Laboratory have provided evidence that C-wMPS has specific advantages for some tasks compared to the conventional systems and that the calibration and layout optimization methods are not only available but also have the ability to improve the performance of the system.

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